Which cosmological model — with dark energy or modified FRW dynamics?

Marek Szydłowski

Astronomical Observatory, Jagiellonian University, Orla 171, 30-244 Kraków, Poland
Complex Systems Research Centre, Jagiellonian University, Reymonta 4, 30-059 Kraków, Poland

E-mail: szydlo@oa.uj.edu.pl

Włodzimierz Godłowski

Astronomical Observatory, Jagiellonian University, Orla 171, 30-244 Kraków, Poland

Abstract. Recent measurements of distant type Ia supernovae (SNIa) as well as other observations indicate that our universe is in accelerating phase of expansion. In principle there are two alternative explanation for such an acceleration. While in the first approach an unknown form of energy violating the strong energy condition is postulated, in second one some modification of FRW dynamics is postulated. The both approaches are in well agreement with present day observations which is the manifestation of the degeneracy problem appearing in observational cosmology. We use the Akaike (AIC) and Bayesian (BIC) information criteria of model selection to overcome this degeneracy and to determine a model with such a set of parameters which gives the most preferred fit to the SNIa data. We consider five representative evolutional scenarios in each of groups. Among dark energy proposal the ΛCDM model, CDM model with phantom field, CDM model with topological defect, model with Chaplygin gas, and the model with the linear dynamical equation of state parameter. As an alternative prototype scenarios we consider: brane world Dvali Gabadadze Porrati scenario, brane models in Randall-Sundrum scenario, Cardassian models with dust matter and radiation, bouncing model with the cosmological constant and metric-affine gravity (MAG) inspired cosmological models. Applying the model selection criteria we show that both AIC and BIC indicates that additional contribution arises from nonstandard FRW dynamics are not necessary to explain SNIa. Adopting the model selection information criteria we show that the AIC indicates the flat phantom model while BIC indicates both flat phantom and flat ΛCDM models.

1. Introduction

If we assume that the FRW model with a source in the form of perfect fluid well describes the present evolution of our Universe, then there is only one way of explain the observational fact that the Universe is accelerating [1, 2, 3] — to postulate some gravity source of unknown nature which violates the strong energy condition $\rho_X + 3p_X > 0$ where ρ_X and p_X are energy density and pressure of dark energy, respectively [4, 5]. These different candidates for dark energy were confronted with observations [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. While the most natural candidate for such a type of dark energy is vacuum energy (the cosmological constant Λ), we are still looking for other alternatives because the fine tuning problem with present value of the cosmological constant. In this context it is considered an idea of dynamical form of the equation of state (EOS) or decaying vacuum. As a first approximation one can consider the coefficient in the EOS $w_X \equiv \frac{p_X}{\rho_X}$ in the form linearized around the present epoch with respect to redshift z or the scale factor a: $w(z) = w_0 + w_1 z$ (or $w(z) = w_0 + w_1(1-a)$, where $a = 1$ corresponds to the present epoch). The models with dust matter and such a form of the EOS for dark energy we called the dynamical EOS (DEOS) model. In the special case when $w_1 = 0$ and $w_0 < -1$ we obtain the CDM quintessence model or phantom CDM (PhCDM) model. Another interesting possibility of description dark energy offers a conception of description of dark energy in terms of Chaplygin gas [6]. In this model the equation of state has a form $p_X = -\frac{A}{\rho_X^\alpha}$.

If we consider the FRW dynamics in which dark energy is present, the basic equation determining the evolution has the following form

$$H^2 = \frac{\rho_{eff}}{3} - \frac{k}{a^2}. \tag{1}$$

where $\rho_{eff}(a)$ is effective energy density of noninteracting "fluids", $k = \pm 1, 0$ is the curvature index. Equation (1) can be presented in terms of density parameters

$$\frac{H^2}{H_0^2} = \Omega_{eff}(z) + \Omega_{k,0}(1+z)^2 \tag{2}$$

where $\frac{a}{a_0} = \frac{1}{1+z}$, $\Omega_{eff}(z) = \Omega_{m,0}(1+z)^3 + \Omega_{X,0} f(z)$ and $\Omega_{m,0}$ is the density parameter for the (baryonic and dark) matter scaling like a^{-3}. For $a = a_0$ (the present value of the scale factor) we obtain the constraint $\Omega_{eff,0} + \Omega_{k,0} = 1$.

We assumed that energy density satisfies the conservation condition

$$\dot{\rho}_i = -3H(\rho_i + p_i), \tag{3}$$

for each component of the fluid $\rho_{eff} = \Sigma \rho_i$. Then from eq. (2) we obtain the constraint relation $\Sigma_i \Omega_{i,0} + \Omega_{k,0} = 1$.

All mentioned before directions which are coming toward to the description of dark energy in the framework of standard FRW cosmology can be treated as a representative approaches of explanation of the current Universe in terms of dark energy. In Table 1 we complete all these models together with the dependence of Hubble's function $H = \frac{\dot{a}}{a}$ with respect to the redshift. We also denote the number of a model's free parameters

by d. Note that for the flat model, $\Omega_{k,0} = 0$, the number of the model parameters is equal $d-1$. As the reference model we consider the flat FRW model with $\Lambda = 0$ (the Einstein-de Sitter model with $\Omega_{m,0} = 1$).

Since the discovery of acceleration of the Universe the theoretical and observational cosmology becomes in the state of permanent tension because of opposite aims of investigations. While the theoretical investigations go towards to generalization degree of consideration by adding some new model parameters, the observational cosmology tries to constraint these parameters. The main goal of observational cosmology is to find a model with a minimal number of essential model parameters. Then this, (of course it may be a naïve model) is a starting point for the further analysis of constraints from the observational data. In the present observational cosmology such a role plays the concordance ΛCDM model and the PhCDM model [19, 20, 21, 22].

Because nature of dark energy is unknown, it is considered another theoretical possibility that the phenomenon of accelerated expansion is actually a sign of a breakdown of the classical Friedmann equation which governs the expansion rate. In this context dark energy effects can be manifestation of a modification to the FRW equation arising from the new exotic physics.

According to the brane world idea, the standard particles are confined on a hypersurface which is called a brane, which is embedded in a higher dimensional bulk space time in which gravity could spread [23]. Then some additional contribution to the standard FRW equation may arise as a consequence of embedding our Universe in the higher dimensional bulk space. It is interesting that the cosmological models formulated in the framework of brane induced gravity can explain the acceleration of the Universe without the conception of dark energy. The additional term in the FRW equation drives the acceleration of the Universe at a late epoch when it is dominant [24, 25, 26, 27, 28]. Therefore the effect arising from existence of these additional dimensions can mimic dark energy through a modified FRW dynamics. As a prototype of evolutional scenarios arising from possible extra dimensions we consider two models: the Dvali Gabadadze Porrati (DGP) model and the Randall-Sundrum scenario of the brane-world (RSB) model. In the DGP model there is present a certain crossover scale r_c that defines what kind of gravity an observer located on the brane observes. While for shorter then r_c distances observer measures the standard gravitational force, for larger then r_c distances gravitational force behaves like r^{-3}. In the RSB models there are present some additional parameters which are absent in the standard cosmology, namely brane tension λ and dark radiation U (see Table 2). If we assume the quantum nature of the Universe then quantum gravity corrections are important at both big bang and big rip singularities. Note that a big rip singularities can not be generic future state in phantom cosmology. Only if energy density is unbounded function of time one obtains a big rip/smash [29]. To account of quantum effects allows to avoid the initial singularity which phenomenologically can be modelled by a bounce [30, 31, 32, 33, 34]. We consider bouncing models as a prototype of evolutional scenario in which quantum effect was important in its very early evolution.

case	name of model	$H(z)$	free parameters	d
0	Einstein-de Sitter	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2}$	$H_0, \Omega_{m,0}$	2
1	ΛCDM	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_\Lambda}$	$H_0, \Omega_{m,0}, \Omega_\Lambda$	3
2	TDCDM	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{T,0}(1+z)}$	$H_0, \Omega_{m,0}, \Omega_{T,0}$	3
3a	PhCDM, $w = -\frac{4}{3}$	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{Ph,0}(1+z)^{3(1+w)}}$	$H_0, \Omega_{m,0}, \Omega_{Ph,0}$	3
3b	PhCDM, w − fitted		$H_0, \Omega_{m,0}, \Omega_{Ph,0}, w$	4
4a	ChGM, $\alpha = 1$	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{Ch,0}\left(A_s + (1-A_s)(1+z)^{3(1+\alpha)}\right)^{\frac{1}{1+\alpha}}}$	$H_0, \Omega_{m,0}, \Omega_{Ch,0}, A_s$	4
4b	GChGM, α − fitted		$H_0, \Omega_{m,0}, \Omega_{Ch,0}, A_s, \alpha$	5
5a	$DEQS$, $p_X = (w_0 + w_1 z)\rho_X$	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{X,0}(1+z)^{3(w_0-w_1+1)}e^{3w_1 z}}$	$H_0, \Omega_{m,0}, \Omega_{X,0}, w_0, w_1$	5
5b	$DEQS$, $p_X = (w_0 + (1-a)w_1)\rho_X$	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{X,0}(1+z)^{3(w_0+w_1+1)}e^{\frac{-3w_1 z}{1+z}}}$	$H_0, \Omega_{m,0}, \Omega_{X,0}, w0, w_1$	5

Table 1. The five prototypes models explaining acceleration in terms of dark energy conceptions.

case	name of model	$H(z)$	free parameters	d
1	DGP	$H = H_0\sqrt{\left(\sqrt{\Omega_{m,0}(1+z)^3+\Omega_{rc,0}}+\sqrt{\Omega_{rc,0}}\right)^2+\Omega_{k,0}(1+z)^2}$	$H_0, \Omega_{m,0}, \Omega_{rc,0}$	3
2a	RSB, $\Lambda = 0$	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3+\Omega_{k,0}(1+z)^2+\Omega_{dr,0}(1+z)^4+\Omega_{\lambda,0}(1+z)^6}$	$H_0, \Omega_{m,0}, \Omega_{\lambda,0}, \Omega_{dr,0}$	4
2b	RSB, $\Lambda \neq 0$	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3+\Omega_{k,0}(1+z)^2+\Omega_{dr,0}(1+z)^4+\Omega_{\lambda,0}(1+z)^6+\Omega_\Lambda}$	$H_0, \Omega_{m,0}, \Omega_{\lambda,0}, \Omega_{dr,0}, \Omega_\Lambda$	5
3	Cardassian	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3+\Omega_{r,0}(1+z)^4+\Omega_{k,0}(1+z)^2+\Omega_{CC,0}}$	$H_0, \Omega_{m,0}, \Omega_{Car,0}, n$	4
	$(\Omega_{r,0} = 0.0001)$	*where* $\Omega_{CC,0} \equiv \Omega_{Car,0}\left(\Omega_{m,0}(1+z)^3+\Omega_{r,0}(1+z)^4\right)^n$		
4	BΛCDM	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3+\Omega_{k,0}(1+z)^2-\Omega_{n,0}(1+z)^n+\Omega_\Lambda}$	$H_0, \Omega_{m,0}, \Omega_{n,0}, \Omega_\Lambda, n$	5
5a	MAG $\Lambda = 0$	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3+\Omega_{k,0}(1+z)^2+\Omega_{\psi,0}(1+z)^6}$	$H_0, \Omega_{m,0}, \Omega_{\psi,0}$	3
5b	MAG $\Lambda \neq 0$	$H = H_0\sqrt{\Omega_{m,0}(1+z)^3+\Omega_{k,0}(1+z)^2+\Omega_{\psi,0}(1+z)^6+\Omega_\Lambda}$	$H_0, \Omega_{m,0}, \Omega_{\psi,0}, \Omega_\Lambda$	4

Table 2. The five prototypes models explaining acceleration in terms of the modification of the FRW equation.

Freese and Lewis [35] have recently proposed to modify the FRW equation by adding a priori an additional term proportional to ρ_{eff}^n (dubbed the Cardassian term by the authors). If we consider a single fluid with energy density ρ then the Cardassian model stays equivalent to a two component noninteracting model. In the special case of dust matter there is a simple interpretation for the origin of this new "Cardassian term"—a perfect fluid satisfying the equation of state $p = (n-1)\rho$. Therefore, if $n < 0$ the phantom cosmological models can be recovered. In our analysis we consider two fluids, matter and radiation. However, note that the radiation term is presently small with comparison to the matter term. We can assume $\Omega_{r,0} = 0.0001$ for radiation matter [36].

The next possibility is the MAG cosmological model based on a non-Riemannian gravity theory [37, 38]. Because this model unifies both the RSB model with vanishing dark radiation and the model with spinning fluid [39] we include this model to the class of models which are based on the modified FRW equation. Because the RSB and MAG models without the cosmological constant do not fit well the SNIa data [10, 38, 40] we also analyze the MAG and RSB models with the non-vanishing Λ term. The five models based on the modified FRW equation are presented in Table 2.

Main goal of this paper is comparison of two conceptually different classes of models using the information criteria of models selection [41]. The Bayesian information criterion gives important information whether additional parameters introduced by "new physics" are actually relevant and to have impact on the current Universe.

2. Distant supernovae as cosmological probes dark energy or the modified FRW equation

Distant type Ia supernovae surveys allowed us to find that the present Universe is accelerating [1, 2]. Every year new SNIa enlarge available data sets with more distant objects and lower systematics errors. Riess et al. [3] compiled the latest samples which become the standard data sets of SNIa. One of them, the restricted "Gold" sample of 157 SNIa, is used in our analysis.

For the distant SNIa one can directly observe their the apparent magnitude m and redshift z. Because the absolute magnitude $\mathcal{M}$ of the supernovae is related to its absolute luminosity L, then the relation between the luminosity distance d_L and the observed magnitude m and the absolute magnitude M has the following form

$$m - M = 5\log_{10} d_L + 25. \tag{4}$$

Instead of d_L, the dimensionless parameter D_L

$$D_L = H_0 d_L \tag{5}$$

is usually used and then eq. (4) changes to

$$\mu \equiv m - M = 5\log_{10} D_L + \mathcal{M} \tag{6}$$

where

$$\mathcal{M} = -5\log_{10} H_0 + 25. \tag{7}$$

We know the absolute magnitude of SNIa from the light curve. The luminosity distance of supernova can be obtain as the function of redshift

$$d_L(z) = (1+z)\frac{c}{H_0}\frac{1}{\sqrt{|\Omega_{k,0}|}}\mathcal{F}\left(H_0\sqrt{|\Omega_{k,0}|}\int_0^z \frac{dz'}{H(z')}\right) \tag{8}$$

where $\Omega_{k,0} = -\frac{k}{H_0^2}$ and

$$\mathcal{F}(x) = \begin{cases} sinh(x) & \text{for } k<0 \\ x & \text{for } k=0 \\ sin(x) & \text{for } k>0 \end{cases} \tag{9}$$

Finally it is possible to probe dark energy which constitutes the main contribution to the matter content. It is assumed that supernovae measurements come with the uncorrelated Gaussian errors and in this case the likelihood function $\mathcal{L}$ can be determined from the chi-square statistic $\mathcal{L} \propto \exp(-\chi^2/2)$ where

$$\chi^2 = \sum_i \frac{(\mu_i^{theor} - \mu_i^{obs})^2}{\sigma_i^2}, \tag{10}$$

while the probability density function of cosmological parameters is derived from Bayes' theorem [1]. Therefore, we can perform the estimation of model parameters using the minimization procedure, based on the likelihood function.

In the modern observational cosmology there is present so called the degeneracy problem: many models with dramatically different scenarios are in good agreement with the present data observations. Information criteria of the model selection [41] can be used to solve this degeneracy among some subclass of dark energy models. Among these criteria the Akaike information (AIC) [42] and the Bayesian information criteria (BIC) [43] are most popular. From these criteria we can determine the number of the essential model parameters providing the preferred fit to the data.

The AIC is defined in the following way [42]

$$AIC = -2\ln\mathcal{L} + 2d \tag{11}$$

where $\mathcal{L}$ is the maximum likelihood and d is a number of the model parameters. The best model with a parameter set providing the preferred fit to the data is that minimizes the AIC.

The BIC introduced by Schwarz [43] is defined as

$$BIC = -2\ln\mathcal{L} + d\ln N \tag{12}$$

where N is the number of data points used in the fit. While AIC tends to favour models with large number of parameters, the BIC the more strongly penalizes them, so BIC provides the useful approximation to full evidence in the case of no prior on the set of model parameters [44].

The effectiveness of using these criteria in the current cosmological applications has been recently demonstrated by Liddle [41] who, taking CMB WMAP data [45], found the number of essential cosmological parameters to be five. Moreover he obtained the important conclusion that spatially-flat models are statistically preferred to close models as it was indicated by the CMB WMAP analysis (their best-fit value is $\Omega_{tot,0} \equiv \Sigma_i \Omega_{i,0} = 1.02 \pm 0.02$ at 1σ level).

In the paper of Parkinson et al. [44] the usefulness of Bayesian model selection criteria in the context of testing for double inflation with WMAP was demonstrated. These criteria was also used recently by us to show that models with the big-bang scenario are rather prefers over the bouncing scenario [46].

Please note that both information criteria values have no absolute sense and only the relative values between different models are physically interesting. For the BIC a difference of 2 is treated as a positive evidence (6 as a strong evidence) against the model with larger value of BIC [47, 48]. Therefore one can order all models which belong to the ensemble of dark energy models following the AIC and BIC values. If we do not find any positive evidence from information criteria the models are treated as a identical and eventually additional parameters are treated as not significant. Therefore the information criteria offer the possibility of introducing relation of weak order in the considered class of analyzed models.

The results of calculation of the AIC and BIC in the context of dark energy models are presented in Tables 3-6. In Table 3 we show results for dark energy models considered for both flat and non flat cases without any assumed extra priors, while in Table 4 we presented results for models with the modified FRW equation. In general case the number of essential parameters in the cosmological models with dark energy is in principal two, i.e., H_0, $\Omega_{m,0}$. It means that the flat model is favored in the light of the information criteria. We can observe two rival models which minimize the AIC and BIC. They are the ΛCDM model and the phantom CDM (PhCDM) model. One can observe that both BIC and AIC values assume lower values for phantom models. It can be regarded as a positive evidence in favor of the PhCDM model. Basing on these simple and objective information criteria we obtain that SNIa data favored the models with the initial (big bang) and final (big rip) singularities.

At first we analyze three flat models with two free parameters, i.e., the flat ΛCDM, TDCDM and PhCDM models. There is a significant difference between predictions of these models. The ΛCDM model prefers a universe with $\Omega_{m,0}$ close to 0.3, the PhCDM model favors a high density universe while the TDCDM model favors a low density universe. In Fig. 1 and Fig. 2 we present values of the AIC and BIC for these and Cardassian models. If $\Omega_{m,0} < 0.22$ then the information criteria favor the TDCDM model. For $\Omega_{m,0} \in (0.22, 0.34)$, the ΛCDM is favored while for $\Omega_{m,0} > 0.34$ the PhCDM model is preferred. The AIC allows also the Chaplygin gas model. However the BIC again prefers the ΛCDM model against the Chaplygin gas model.

The similar analysis with the use of the information criteria is done in the case of the assumed prior $\Omega_{m,0} = 0.3$ [49] (Table 4). In this case the model with Λ is preferred over

the models with phantoms, that is in contrary to the results obtained in the previous case of no priors for $\Omega_{m,0}$. It clearly shows that more precise measurements of $\Omega_{m,0}$ will give us the possibility to discriminate between the ΛCDM and PhCDM models.

For the model models with the modified FRW equation information criteria prefferes only the Cardassian model. However this model is preferred in the case of a high density universe, with $\Omega_{m,0} > 0.44$ which seems to be too high with comparison with present extragalactic data [49].

The comparison of PhCDM models with different fixed value of w_X shows Fig. 3 and Fig. 4. Note that it is required to have higher matter content to equilize the greater negative effect of lower (more negative) value of the factor w. The model which minimize the values of both AIC and BIC is the model with $w = -2.37$.

Fig. 5 and Fig. 6 shows the AIC and BIC in respect to $\Omega_{m,0}$, for low density universes (the TDCDM and DGP models) and the ΛCDM model. Dependent on value of $\Omega_{m,0}$ the different cosmological models are selected. We find that TDCDM model is distinguished for $\Omega_{m,0} < 0.16$, DGP model for $\Omega_{m,0} \in (0.16, 0.24)$ and ΛCDM for $\Omega_{m,0} > 0.24$.

For deper analysis of statistical results it would be useful to consider the information entropy of the distribution which is defined as

$$Entropy = -\Sigma f_i \log_a(f_i) \qquad (13)$$

where a is a number of independent states of the system.

In Table 7 the value of entropy for four flat models (with topological defect, with the cosmological constant, with phantom and for the Cardassian model). The value of entropy for one dimensional PDF ($\Omega_{m,0}$) is also presented. We can see that in both cases we obtain minimal value of entropy for the PhCDM model.

Basing on these simple and objective information criteria and the minimum entropy principle we obtain that SNIa data favor the models with dark energy rather than based on modified FRW equations. Among the dark energy models the best candidates are the PhCDM and ΛCDM models. However, to make the final decision which model describes our Universe it is necessary to obtain the precise value of $\Omega_{m,0}$ from independent observations.

3. Conclusion

The main goal of this paper is to decide which class of models: dark energy or models based on the modified FRW equation are distinguished by statistical analysis of SNIa data. For this aims we use the Akaike and Bayesian information criteria.

It was considered two groups of five different models containing dust and dark energy (the first group) and dust matter and an additional term which modifies FRW dynamics (the second group). Our main conclusion is that both criteria weigh in favor of the flat dark energy models. We argue that there is no strong reason to favor models with modified FRW dynamics over FRW dark energy models.

We are also able to decide which model with dark energy is distinguished by the statistical analysis of SNIa data. To do this we use the Akaike and Bayesian information

criteria. The former criterion weighs in favor of the flat phantom model while the latter distinguishes the flat phantom and ΛCDM models. Assuming the prior $\Omega_{m,0} = 0.3$ both the AIC and BIC criteria weighs in favor of the model with dark energy, namely the flat ΛCDM model.

The further conclusions are the following.

- The minimal number of essential parameters in the cosmological models with dark energy is in principal two, i.e., $(H_0, \Omega_{m,0})$. The list of essential parameters may be longer, because some of them are not convincingly measured with present data, like the parameter w_1.
- The curvature density parameter does not belong to the class of essential parameters when all the rest parameters are without any priors (with no fixed $\Omega_{m,0}$). At this point our result coincides with analogous result obtained by Liddle who found it basing on other observations.
- If we consider models in which all model parameters are fitted then the PhCDM model with double initial and final singularities is distinguished.
- When we consider the prior on $\Omega_{m,0}$ then while for $\Omega_{m,0} < 0.16$ the model with two-dimensional topological defect is favored. The value $\Omega_{m,0} \in (0.16; 0.24)$ favor DGP model while the value $\Omega_{m,0} \in (0.22; 0.34)$ favor the ΛCDM model. For $\Omega_{m,0} \in (0.22; 0.34)$ the phantom model is preferred. With high density Universe $(\Omega_{m,0} > 0.44)$ Cardassian model (with modified FRW equation of state) is preferred.

During our analisis we have use recently avaliable SNIa data and of course future SNAP data will provide mor sophisticated information to distinguish between FRW dark energy models and the models with modified FRW dynamics.

To make the ultimate decision which model describes our Universe it is necessary to obtain the precise value of $\Omega_{m,0}$ from independent observations.

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Table 3. The values of the AIC and BIC for the models from Table 1 both for flat and non-flat cases.

case	AIC ($\Omega_{k,0} = 0$)	AIC ($\Omega_{k,0} \neq 0$)	BIC ($\Omega_{k,0} = 0$)	BIC ($\Omega_{k,0} \neq 0$)
0	325.5	194.4	328.6	200.5
1	179.9	179.9	186.0	189.0
2	183.2	180.1	189.4	194.4
3a	178.0	179.3	184.1	188.5
3b	178.5	179.7	187.7	191.9
4a	179.7	181.4	188.9	193.6
4b	181.7	183.4	193.9	198.7
5a	180.5	182.0	192.7	197.3
5b	180.4	181.9	192.6	197.2

Table 4. The values of the AIC and BIC for the models from Table 2 both for flat and non-flat cases.

case	AIC ($\Omega_{k,0} = 0$)	AIC ($\Omega_{k,0} \neq 0$)	BIC ($\Omega_{k,0} = 0$)	BIC ($\Omega_{k,0} \neq 0$)
1	180.9	180.0	187.0	189.1
2a	327.0	184.6	336.2	196.8
2b	182.1	183.8	194.3	199.1
3	178.5	179.7	187.7	191.9
4	183.9	183.6	196.2	198.8
5a	315.1	195.8	321.2	205.0
5b	180.3	181.6	189.4	193.8

Table 5. The values of the AIC and BIC for the models from Table 1 with the prior $\Omega_{m,0} = 0.3$ both for flat and non-flat cases.

case	AIC ($\Omega_{k,0} = 0$)	AIC ($\Omega_{k,0} \neq 0$)	BIC ($\Omega_{k,0} = 0$)	BIC ($\Omega_{k,0} \neq 0$)
0	—	216.9	—	220.0
1	177.9	179.9	181.0	186.0
2	190.0	178.8	193.0	184.9
3a	183.9	179.6	187.0	186.7
3b	179.9	178.2	186.0	187.4
4a	179.6	179.8	185.7	188.9
4b	181.6	181.8	190.8	194.0
5a	179.2	180.2	188.4	192.4
5b	178.8	180.3	187.9	192.5

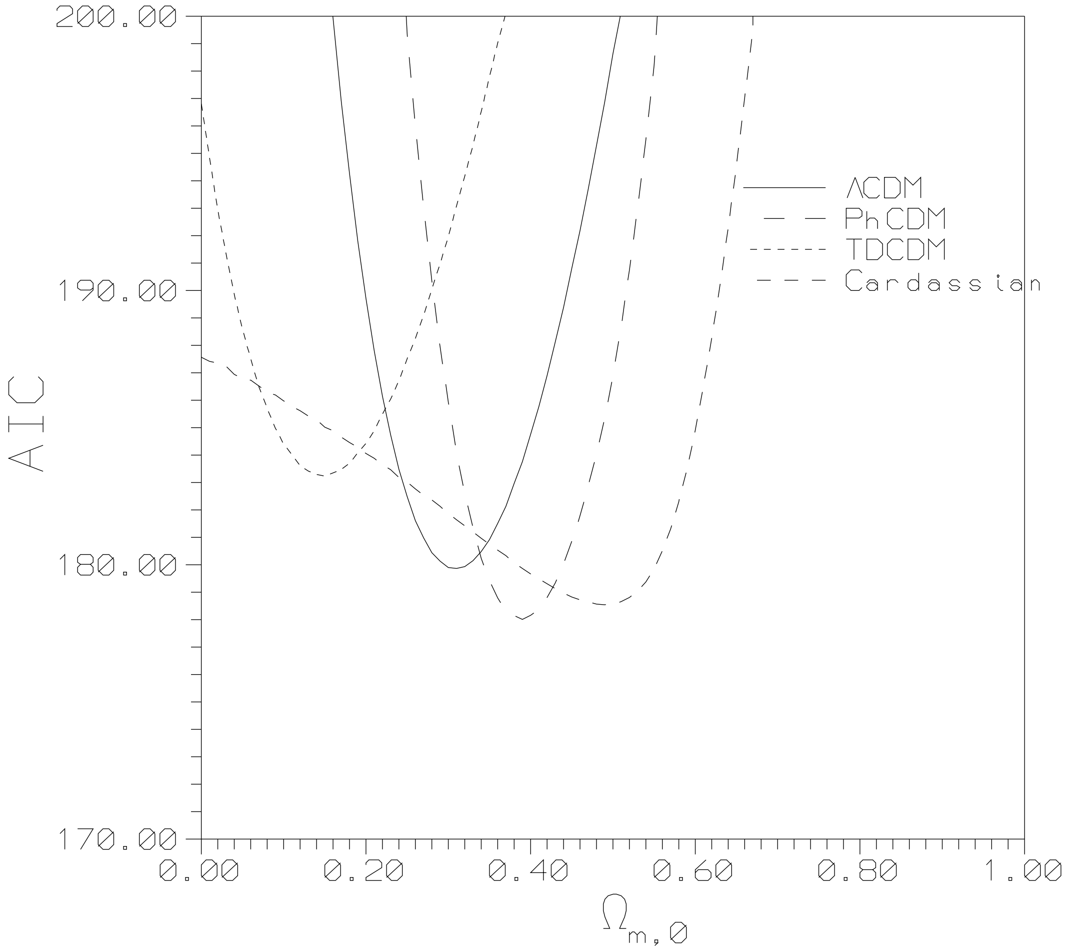

Figure 1. The values of the AIC in respect to fixed value $\Omega_{m,0}$ for four flat models (with topological defect, with the cosmological constant, with phantom and for the Cardassian model).

Table 6. The values of the AIC and BIC for models from Table 2 with the prior $\Omega_{m,0} = 0.3$ both for flat and non-flat cases.

case	AIC ($\Omega_{k,0} = 0$)	AIC ($\Omega_{k,0} \neq 0$)	BIC ($\Omega_{k,0} = 0$)	BIC ($\Omega_{k,0} \neq 0$)
1	185.9	178.2	189.0	184.3
2a	474.7	183.0	480.9	192.2
2b	180.5	181.9	189.7	194.1
3	179.9	178.2	186.0	187.4
4	181.9	183.9	191.1	196.1
5a	1296.3	204.4	1299.3	210.5
5b	179.5	180.1	185.6	189.3

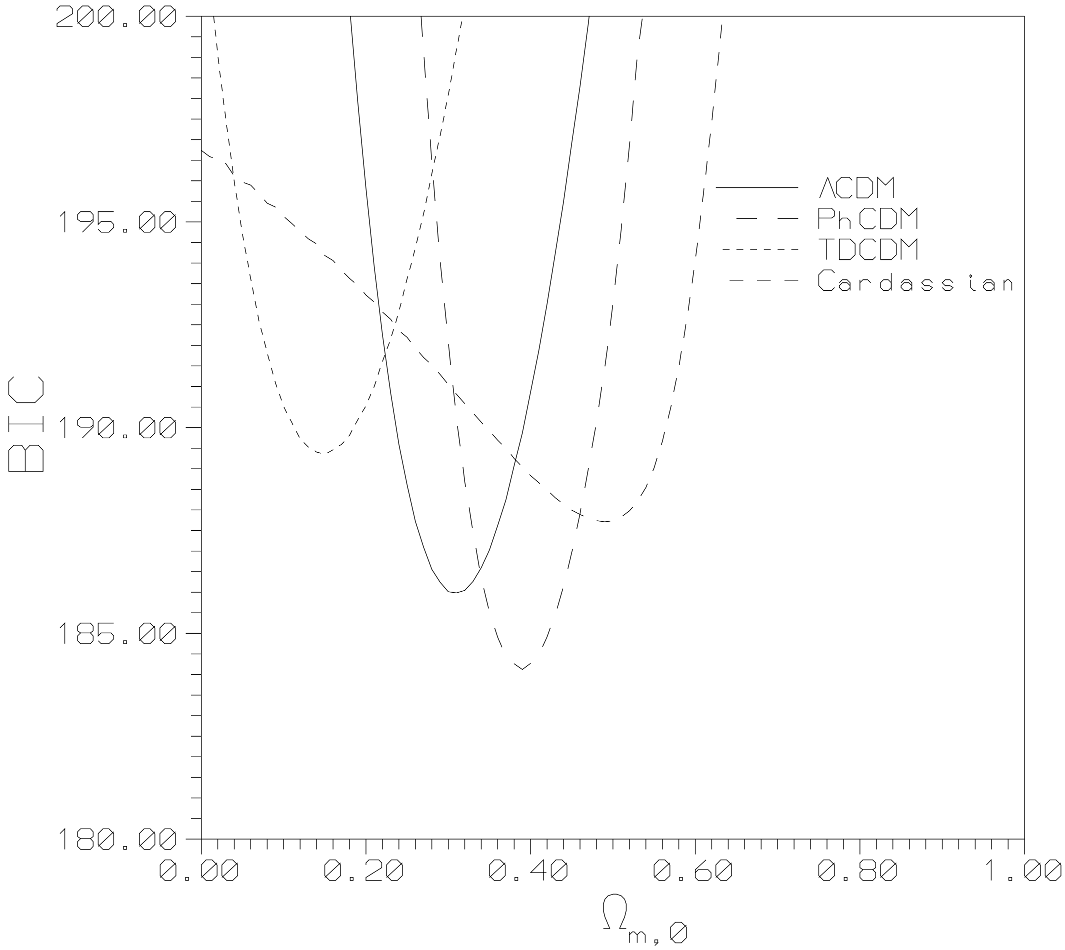

Figure 2. The values of the BIC in respect to fixed value $\Omega_{m,0}$ for four flat models (with topological defect, with the cosmological constant, with phantom and for the Cardassian model).

Table 7. The value of information entropy for four flat models (with topological defect, the cosmological constant, phantom and for the Cardassian model). The value of entropy for the one dimensional PDF of $\Omega_{m,0}$ is also presented.

model	entropy	entropy ($\Omega_{m,0}$)
ΛCDM	0.604	0.601
TdCDM	0.627	0.641
PhCDM	0.591	0.577
Cardassian	0.718	0.682

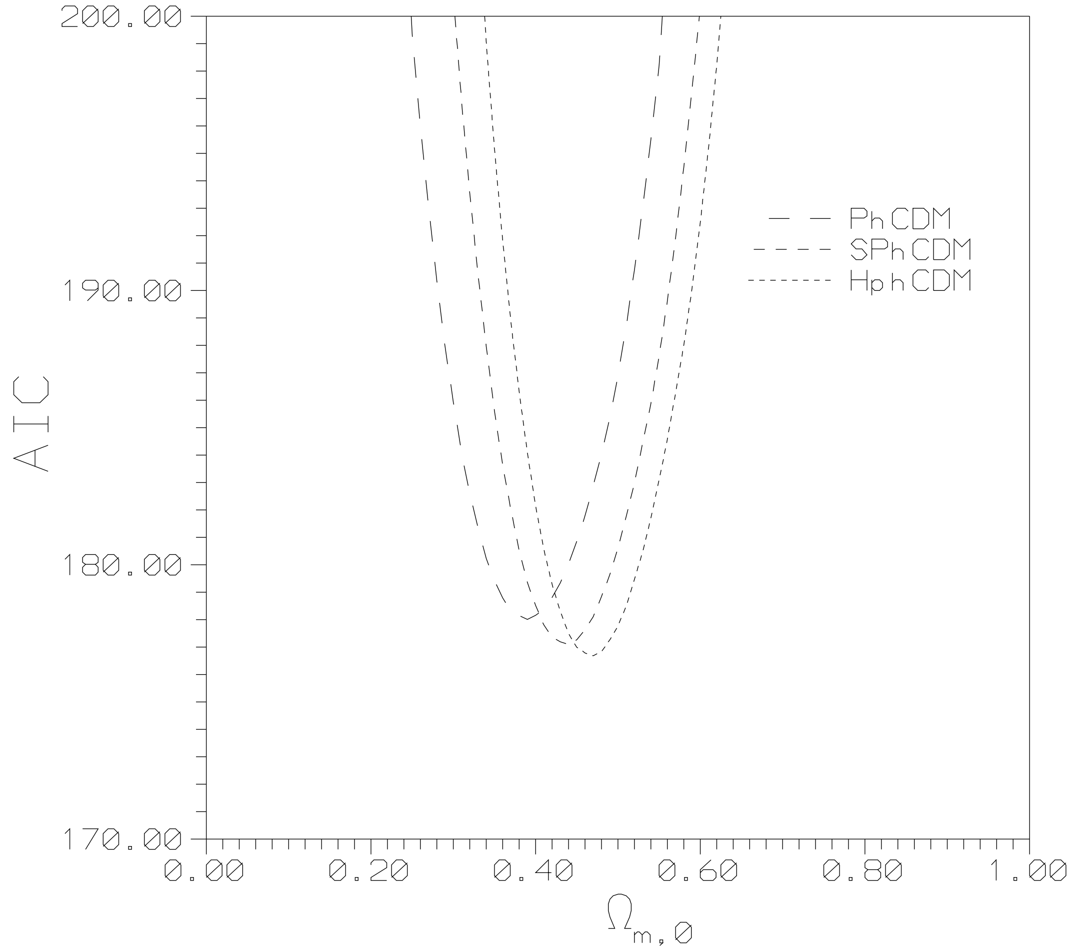

Figure 3. The values of the AIC in respect to fixed value $\Omega_{m,0}$ for three flat phantom, super nad hiper phantoms models with different fixed value of w - PhCDM ($w = -4/3$), SPhCDM ($w = -5/3$), HPhCDM ($w = -2$).

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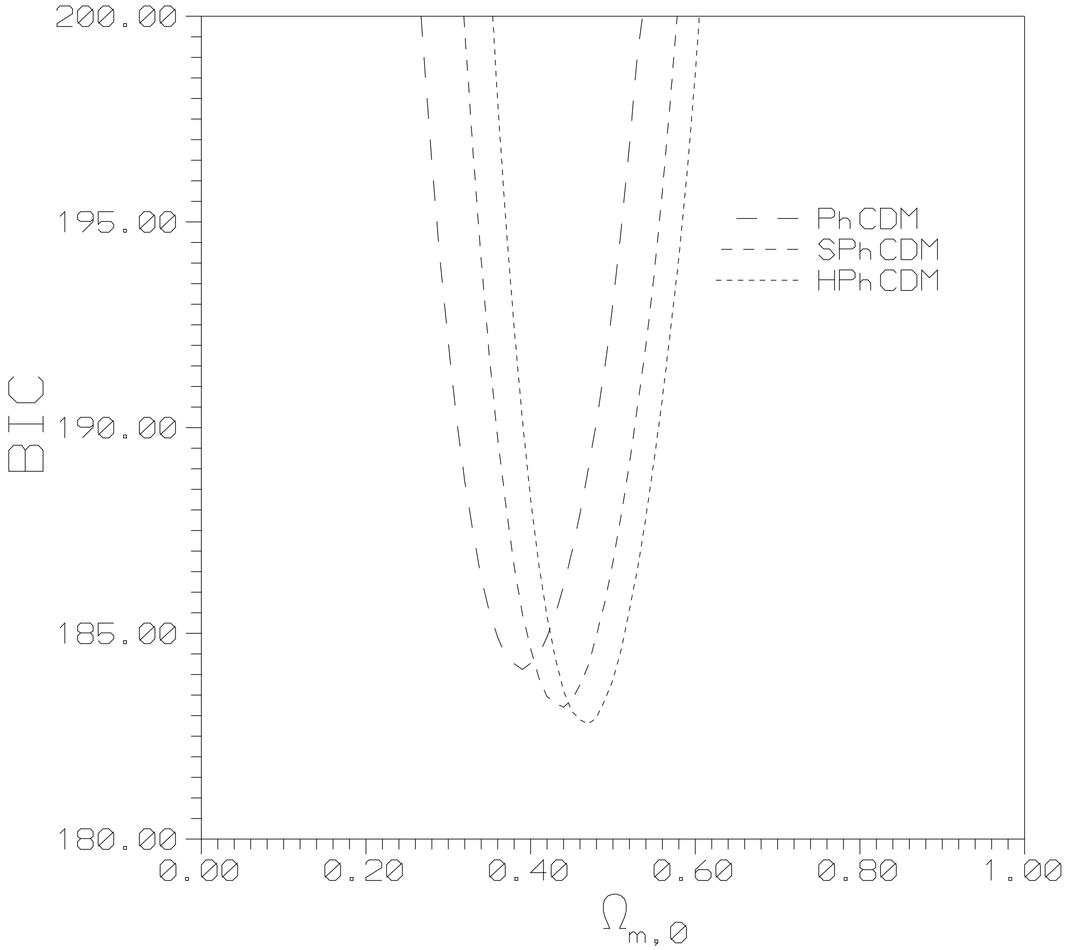

Figure 4. The values of the BIC in respect to fixed value $\Omega_{m,0}$ for three flat phantom, super nad hiper phantoms models with different fixed value of w - PhCDM ($w = -4/3$), SPhCDM ($w = -5/3$), HPhCDM ($w = -2$).

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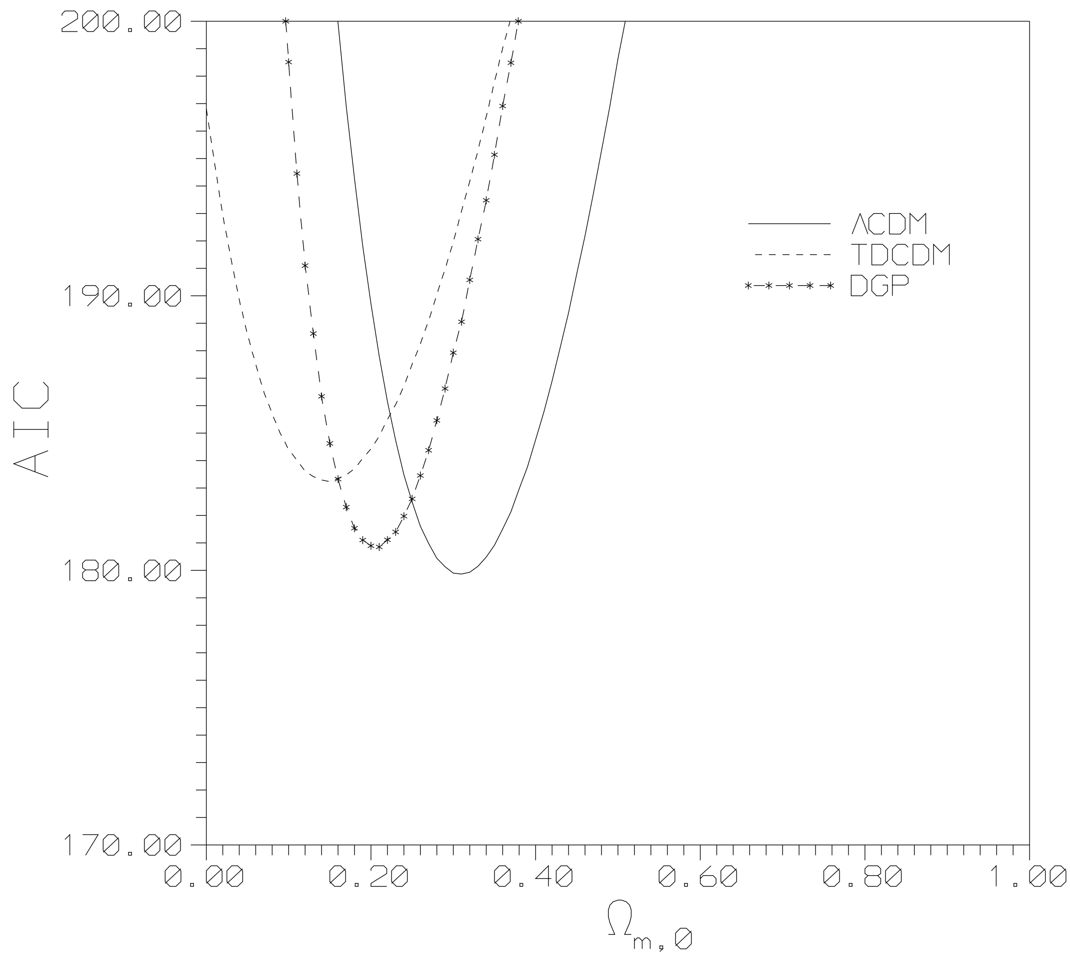

Figure 5. The values of the AIC in respect to fixed value $\Omega_{m,0}$ for low density (TDCDM and DGP) models and ΛCDM model.

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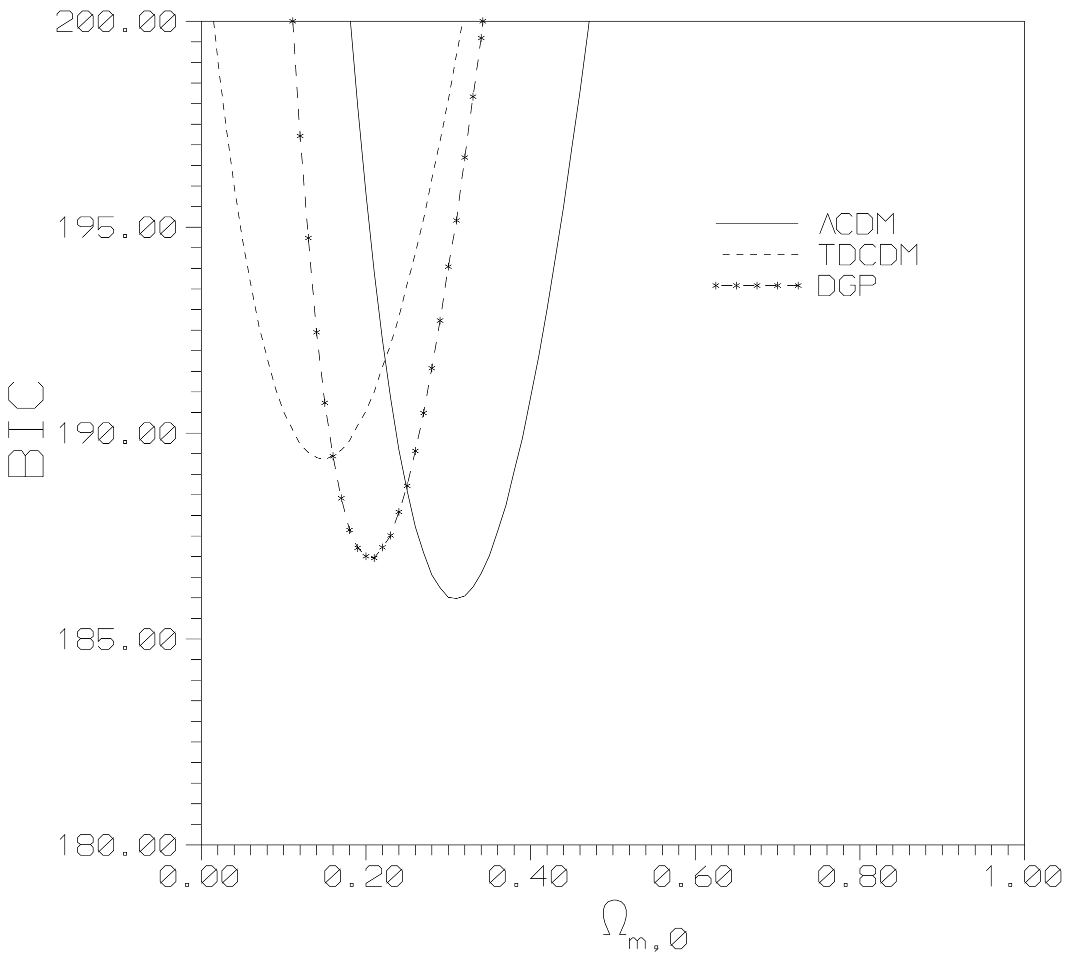

Figure 6. The values of the BIC in respect to fixed value $\Omega_{m,0}$ for low density (TDCDM and DGP) models and ΛCDM model.